AI-1526-M.E.-CV-19

M.A./M.Sc. - (Previous)

Term End Examination, Mar.-Apr.-2021

MATHEMATICS

Differential Geometry of Manifolds, Paper-IV

Time : Three Hours] [Maximum Marks : 100

Note : Answer any Five Question. All Question carry equal marks.

1. (a) Define differentiable manifold and show that the real projective space PR^n is differentiable manifold.
   (b) State and prove Local immersion theorem.

2. Let (M, g) be a Riemann manifold with sectional curvature $k \geq k_0 > 0$. Then show that for any geodesic c in M, the distance between two conjugate points along c is $\leq \frac{\pi}{\sqrt{k_0}}$

3. (a) Show that the tangent bundle of a Lie group is trivial $TG \cong G \times g$.
   (b) Show that the range of the zero section of a vector bundle $E \to M$ is a submanifold of E.

4. (a) State and prove Schur's Theorem.
   (b) Define the following with example:-
      (i) Nijenhuis tensor
      (ii) Conformal curvature tensor
      (iii) Exterior derivative
      (iv) Bundle homomorphim

5. Let $g: L \to Q$ be a surjective submersion which is proper, show that $g^{-1}(k)$ is compact in L for each compact $k \subset Q$ and let Q be connected. Then show that (L, P, Q) is fibre bundle.

6. State and prove Generalized Gauss and Mainardi-Codazzi equations.

7. Show that the circle $s' \subset \not\subset$ is a Lie group under complex multiplication and the map

$$z = e^{i\theta} \to \begin{bmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & I_{n-2} \end{bmatrix}$$

   Is a Lie group homomorphism into $SO(n)$.

8. (a) Prove that every vector bundle of dimension n over V is associated to a principal bundle over V with group $GL(n, R)$.
   (b) Define the following:-
      (i) Tangent bundle
      (ii) Induced bundle
      (iii) Principle fibre bundle

9. (a) Prove that Riemannian geodesic is Locally minimizing.
   (b) State and prove First variation formulae.

10. (a) Show that tangent bundle is a vector Bundle.
    (b) Prove that $X^T = KMOTX$ for vector Bundle Homomorphism.